Chapter 4

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

Previously, we used **only** an x or y direction. What about if something moves in 2 dimensions?

If an object moves in the *xy*-plane with constant acceleration, then both a_x and a_y are constant.

By looking separately at the motion along two perpendicular axes, the *y*-direction and the *x*-direction, each component becomes a one-dimensional problem, which we already know how to solve.

We can apply any of the constant acceleration relationships we previously studied, separately to the *x*-components and to the *y*-components.

The x and y components of a problem can be treated separately and independently. The only common item between the two is typically "time"

Suppose we choose the axes so that the acceleration is only in the positive or negative *y*-direction. Then $a_x = 0$ and v_x is constant. With this choice (not the only one!) then in this example:

x-axis: $a_x = 0$

$$\Delta v_x = 0 \qquad (v_x \text{ is constant})$$

$$\Delta x = v_x \Delta t$$

y-axis: constant a_y

$$\Delta v_y = a_y \Delta t$$

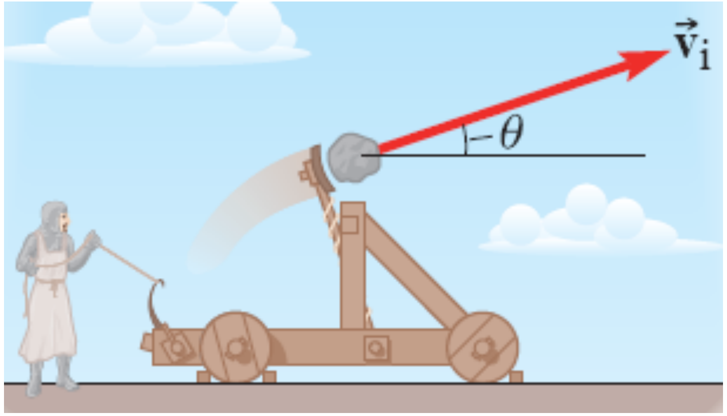
$$\Delta y = \tfrac{1}{2}(v_{fy} + v_{iy})\,\Delta t$$

$$\Delta y = v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{fy}^2 - v_{iy}^2 = 2a_y\,\Delta y$$

Now we consider objects (called **projectiles**) in free fall that have a *nonzero* horizontal velocity component.

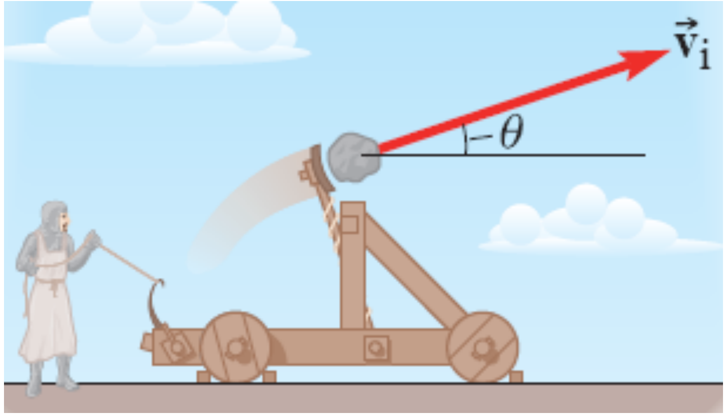
The **angle of elevation (θ)** is the angle of the initial velocity above the horizontal.



Once the stone is in the air, the we assume that gravity acts and can consider the special case where the air resistance has a negligible effect on the motion.

The **trajectory** is the path of the stone.

If the initial velocity $\vec{\mathbf{v}}_i$ is at an angle θ above the horizontal, then resolving it into components gives



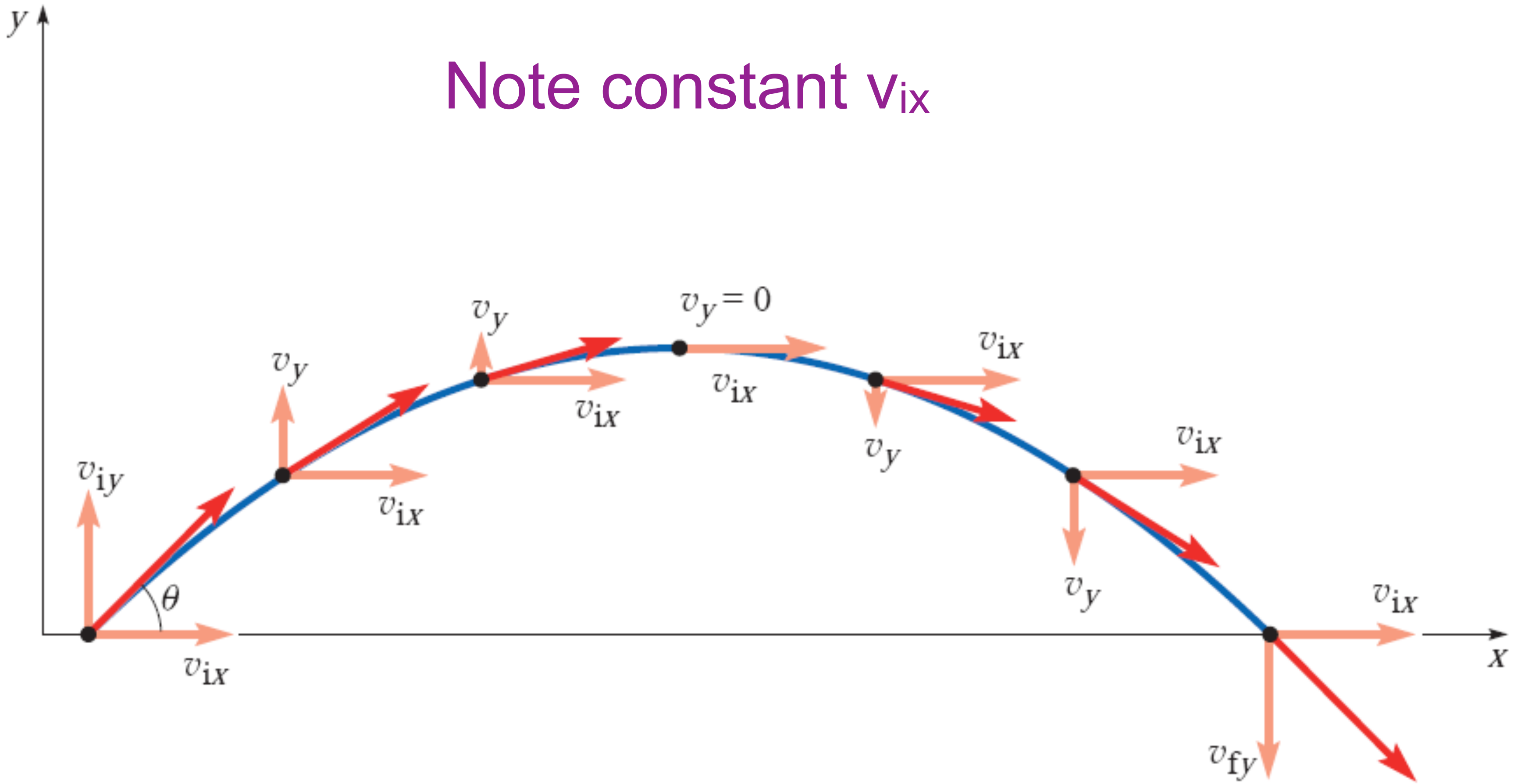
What are the coordinate axes?

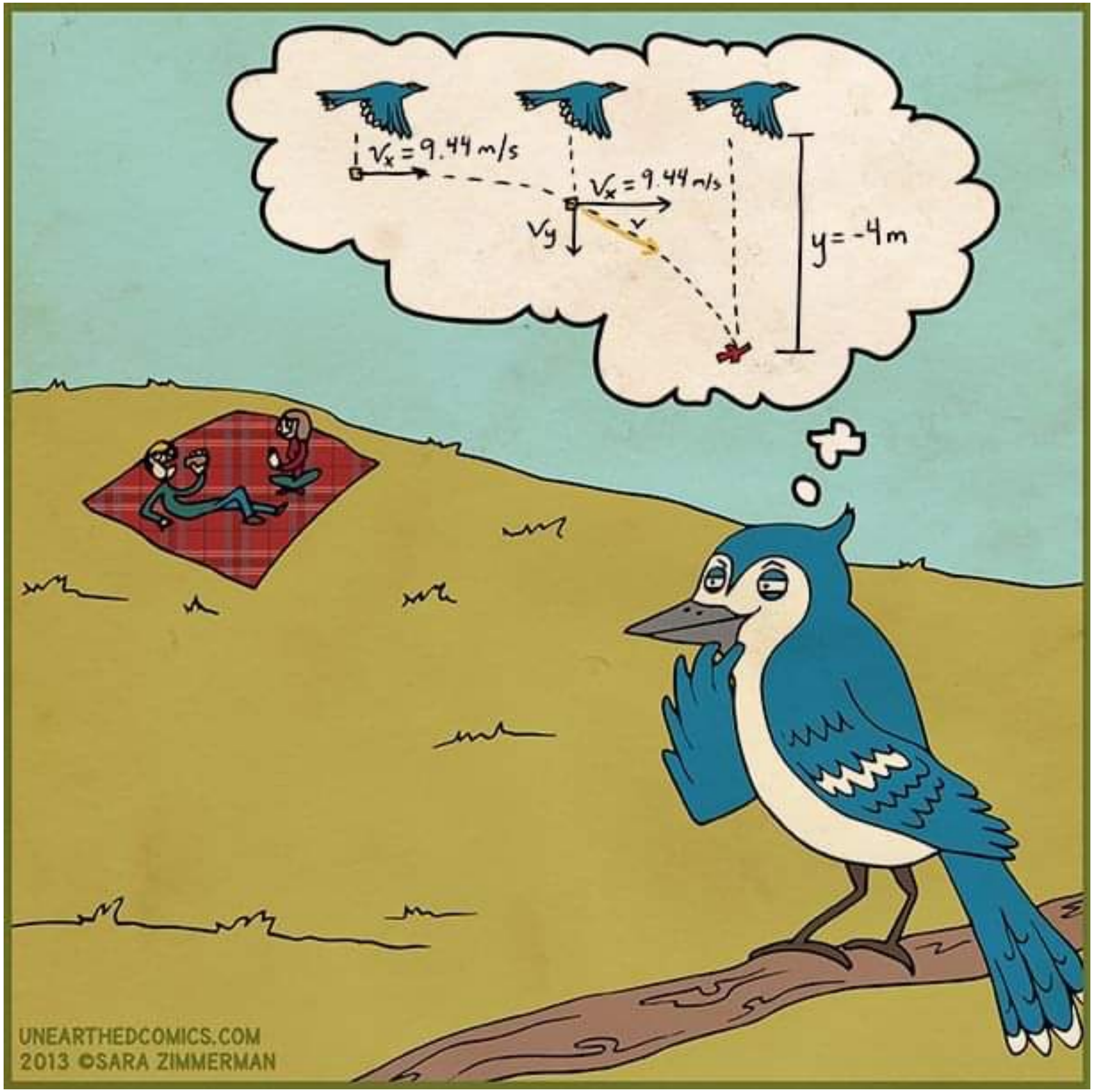
$$v_{ix} = v_i \cos\theta \qquad \text{and} \qquad v_{iy} = v_i \sin\theta$$

(+y-axis up, θ measured from the horizontal x-axis)

$a_x = 0$ and the stone's horizontal velocity component v_x is *constant*

Note constant v_{ix}





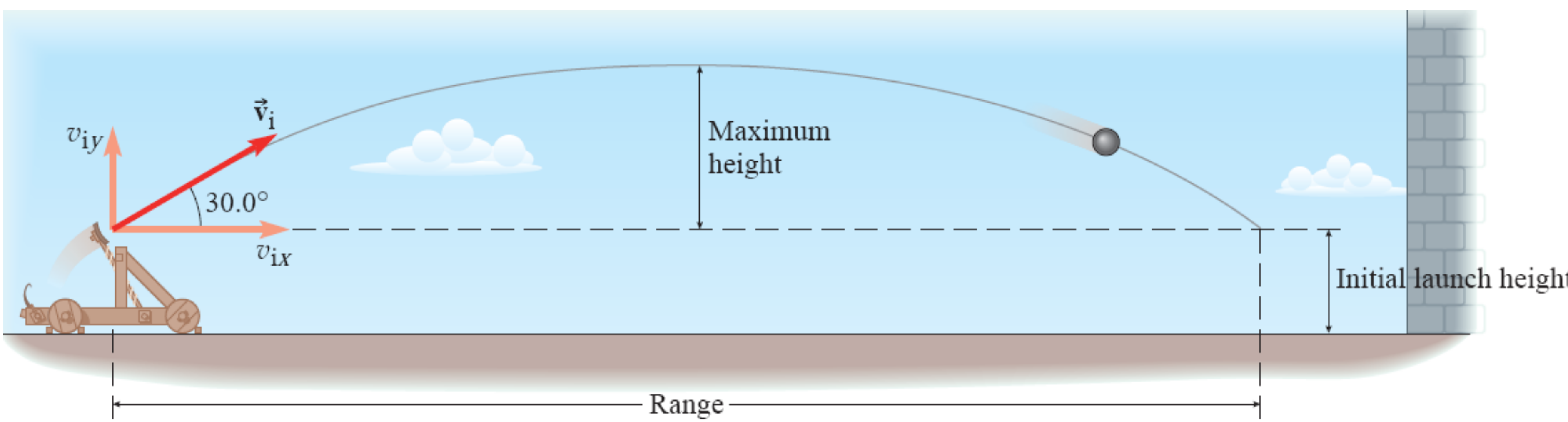
Another way to look at projectiles

The catapult used by the marauders hurls a stone of mass 32.0 kg with a velocity of 50.0 m/s at a 30.0° angle of elevation.

(a) What is the maximum height reached by the stone?

(b) What is its *range* (defined as the horizontal distance traveled when the stone returns to its original height)?

(c) How long has the stone been in the air when it returns to its original height?



Solution

(a)

$$v_{iy} = v_i \sin\theta \quad \text{and} \quad v_{ix} = v_i \cos\theta$$

"final" = at maximum height, v_{fy} = 0. Why?

$$v_{fy}^2 - v_{iy}^2 = 2a_y\,\Delta y$$

a_y = -g
Δy = max height

$$\Delta y = \frac{-(50.0\text{ m/s} \times \sin 30.0°)^2}{2 \times (-9.80\text{ m/s}^2)} = 31.9\text{ m}$$

CAREFUL with minus signs!

The maximum height of the projectile is 31.9 m above its launch height.

Solution

(c) We solve (c) before (b).

$\Delta y =$ $0 = v_{iy}t + \frac{1}{2}a_y t^2$

t = 0 (why?) OR
$0 = v_{iy} + 0.5(-g)t$

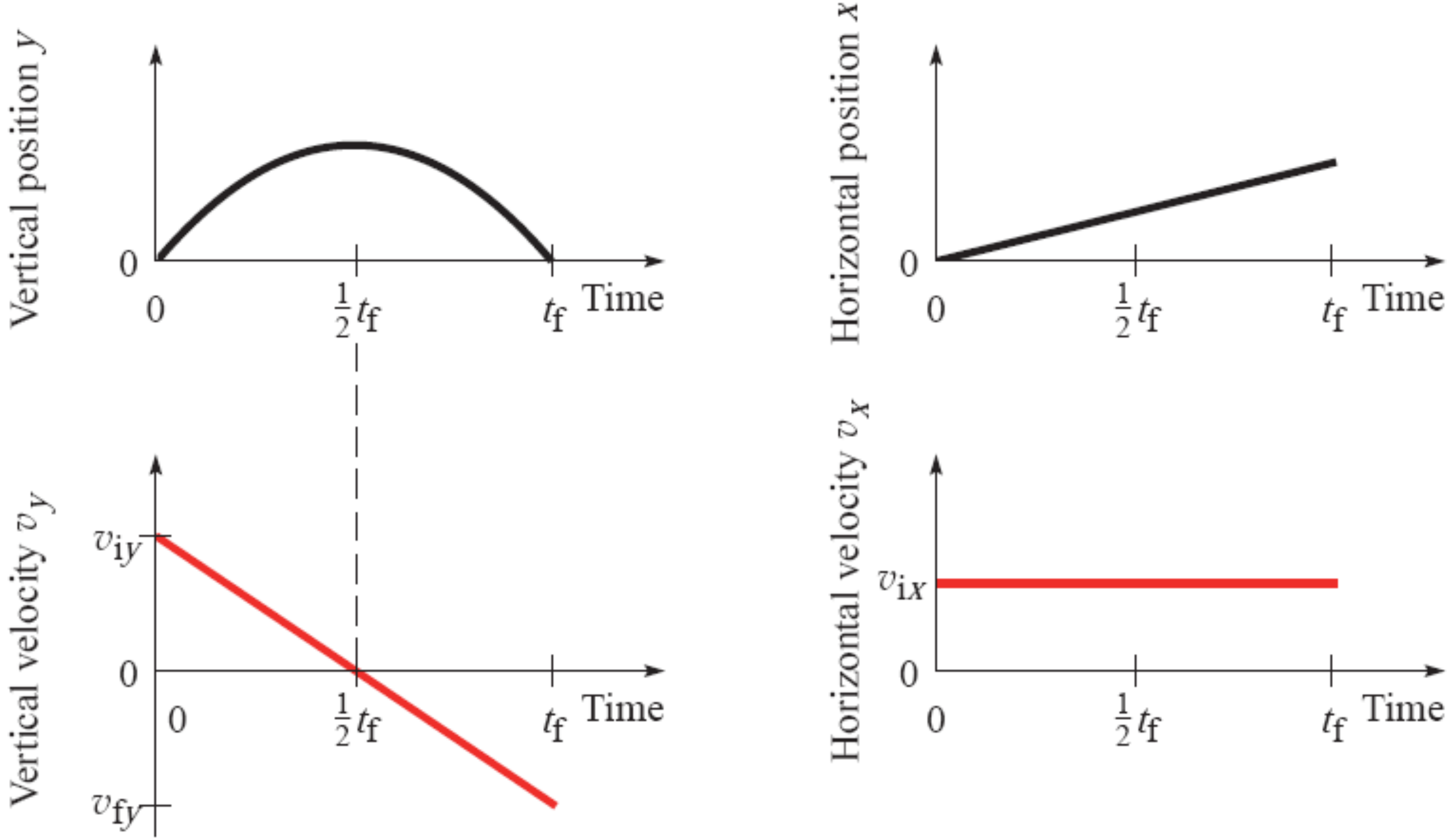
The time of flight is

$t_f =$ $2 \times \dfrac{+50.0 \text{ m/s} \times \sin 30.0°}{+9.80 \text{ m/s}^2} = 5.10 \text{ s}$

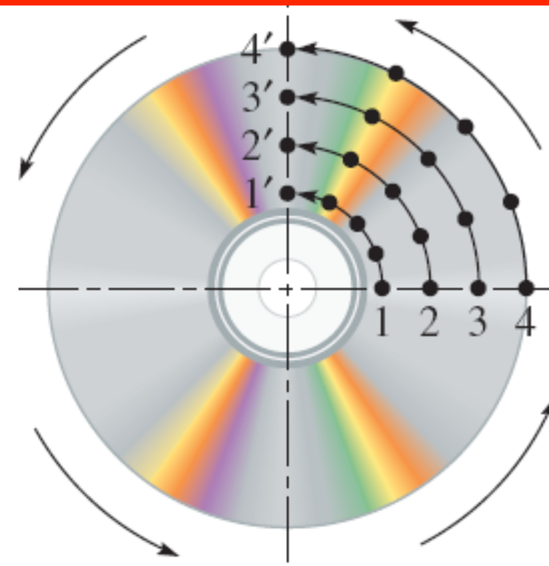
(b) The range is

$$\Delta x = v_{ix}t_f = (50.0 \text{ m/s} \times \cos 30.0°) \times 5.10 \text{ s} = 221 \text{ m}$$

Graphing Projectile Motion



Let's look carefully at this



When a DVD spins inside a DVD player, different points on the DVD have different velocities and accelerations.

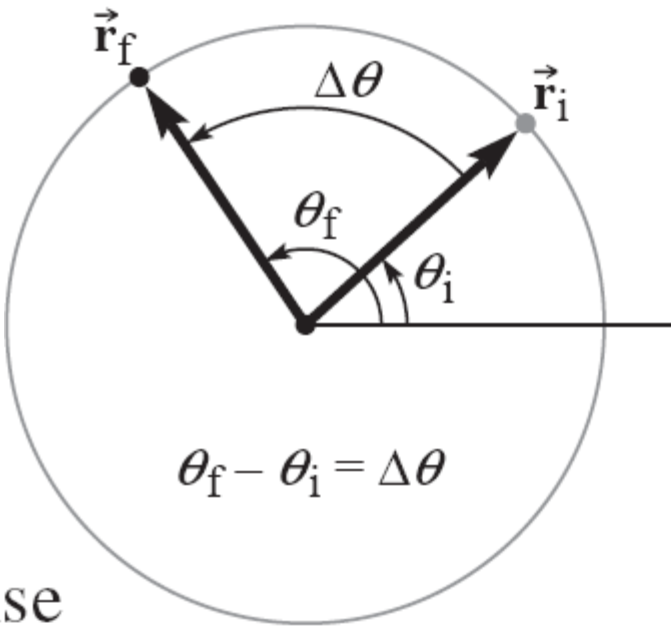
The velocity and acceleration of a given point keep changing direction as the DVD spins.

It is much simpler to say “the DVD spins at 210 rpm” than to say “a point 6.0 cm from the rotation axis of the DVD is moving at 1.3 m/s.”

One point on the CD **does not move.**
Which one?

Angular Displacement and Angular Velocity
To simplify the description of circular motion, we concentrate on *angles* instead of distances.

Instead of displacement, we speak of **angular displacement $\Delta\theta$**, angle through which the DVD turns.

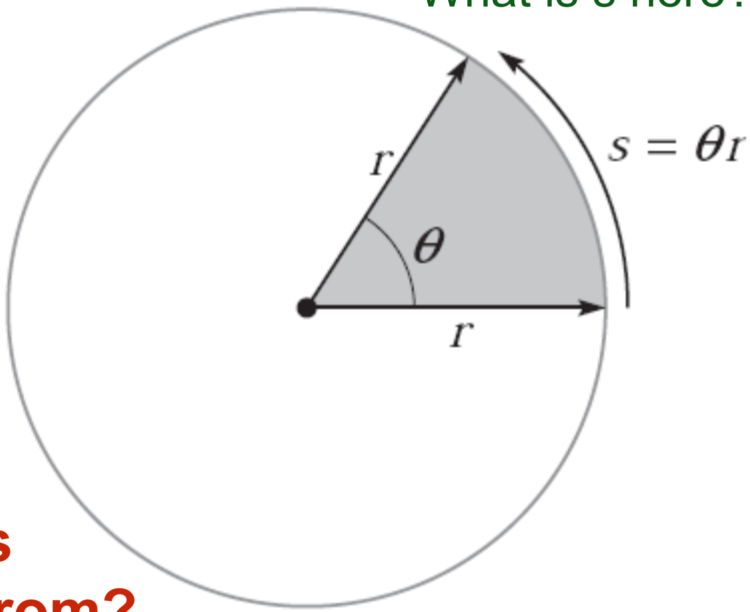


Not our normal usage (as in a clock), so be careful!
Also, what happens if we view the spinning disc from the other side?

+ Counterclockwise

– Clockwise

What is s here?



Radian Measure

In many situations the most convenient angle measure is the **radian**.

$$\theta \text{ (in radians)} = \frac{s}{r}$$

Where does this come from?

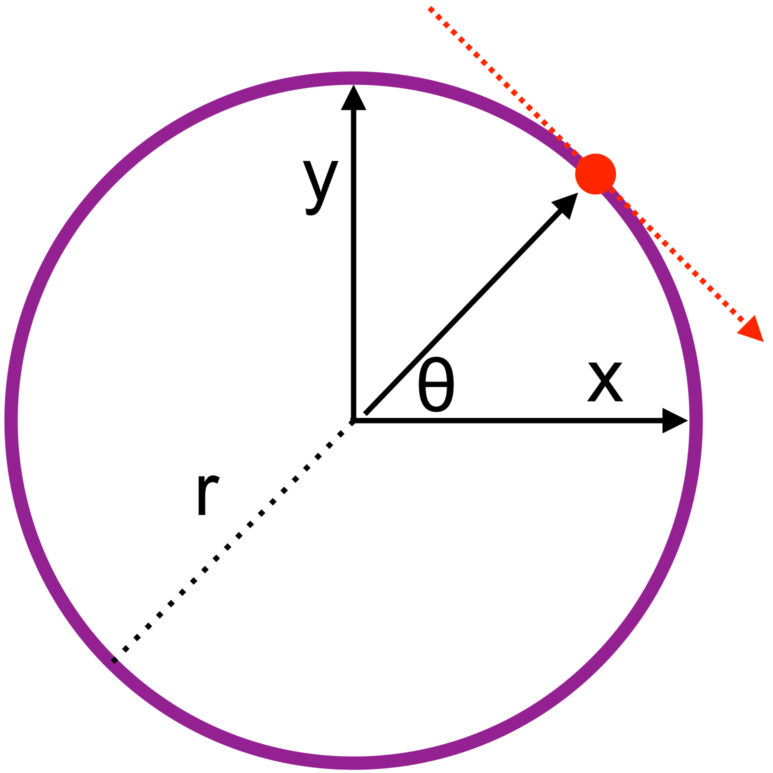
Be careful about which angular units you're using! (Especially on a calculator)

For full circle: $\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$

$$360° = 2\pi \text{ rad}$$

Recall the relationship between x and y for objects moving in a circle in the x-y plane with radius r:

$x^2 + y^2 = r^2$



Do we know of any formulas that might satisfy this?

$\cos^2\theta + \sin^2\theta = 1$

$r^2 * \cos^2\theta + r^2 * \sin^2\theta = r^2$

Or $x = r*\cos(\theta)$

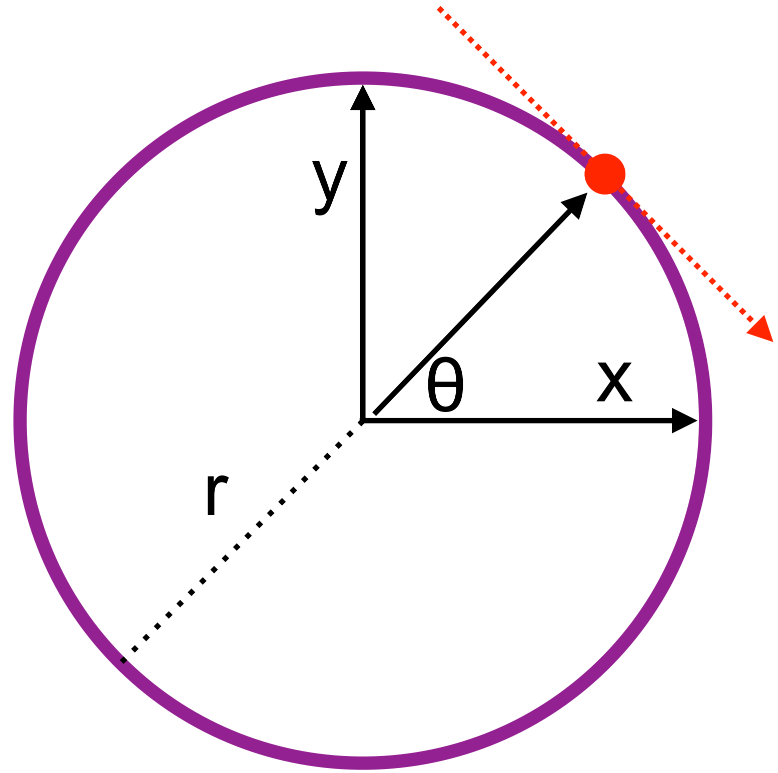
Or $y = r*\sin(\theta)$

$\theta = \tan^{-1}(y/x)$

How does θ change with time?

Define: $\omega = d\theta/dt$ (angular speed is rate of change of angle)

How does θ change with time? Define: $\omega = d\theta/dt$ (angular speed is rate of change of angle). Then: $\theta = \omega t$ (+ constant phase)



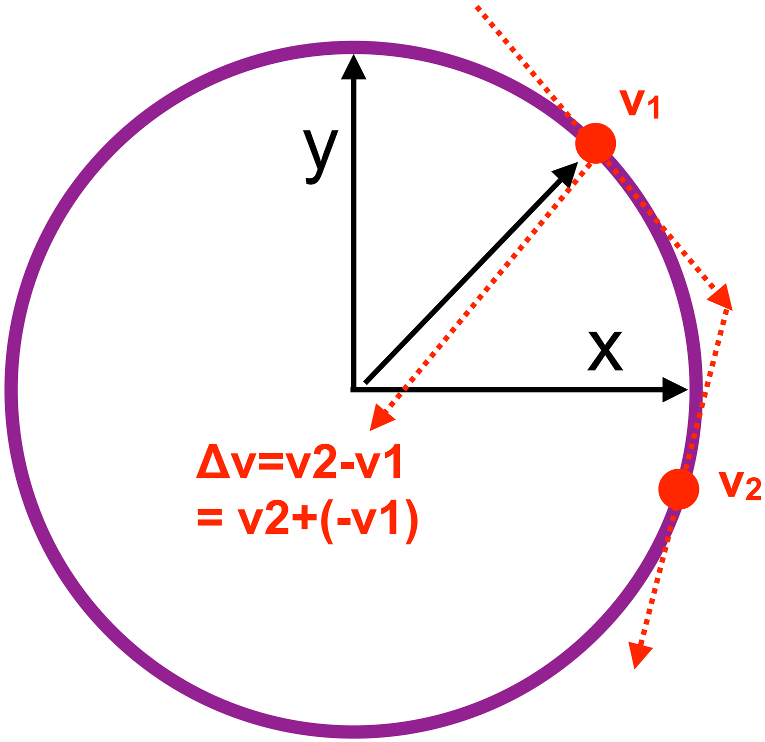
What is the distance traveled as the ball moves around the full circle? Circumference of circle = $2\pi r$. The amount of time this takes is called the **period (T)**

In a full cycle, the ball travels 2π radians. By definition this takes time $t=T$. So: $\omega = 2\pi/T$

For a particle undergoing **uniform circular motion**, the *direction* of the velocity continually changes, so the particle has a nonzero acceleration. **WHY?**

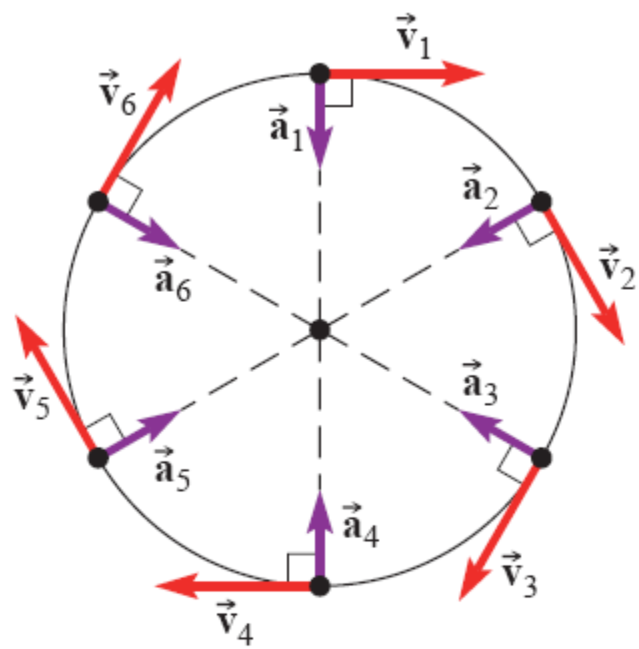
The acceleration **a** has the same direction as Δ**v** (in the limit $\Delta t \rightarrow 0$).

The acceleration is directed radially inward. The acceleration of an object undergoing *uniform* circular motion is called **radial acceleration** (sometimes called ***centripetal acceleration)***



Can't get away from those pesky vectors!

The velocity vector keeps changing, which means the object is constantly accelerating. And the direction of the acceleration is always inwards, so it is constantly changing, too!



Connecting circular and linear motion

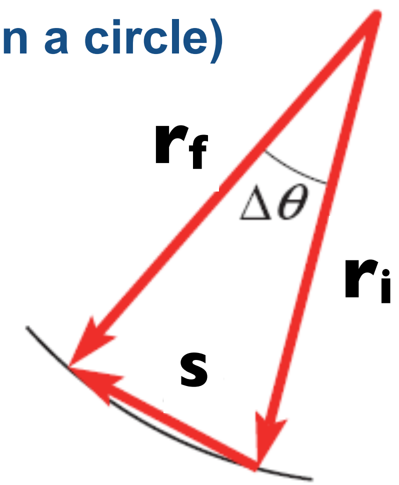
Here only talking about radial acceleration (moving in a circle)

The **linear** speed is as always

$v = ds/dt = d(r\Delta\theta)/dt.$

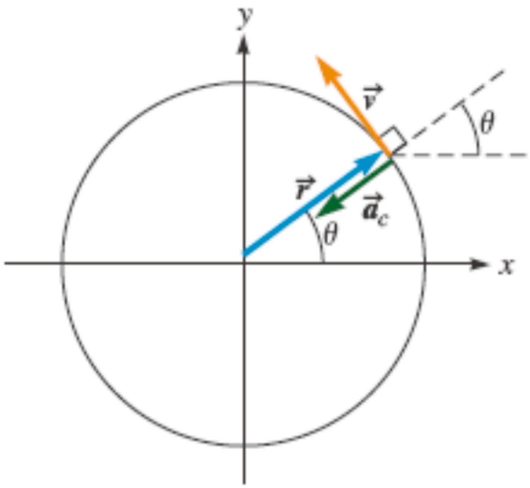
If we move in a circle, r is constant so

$v = r\, d\theta/dt = r\omega$

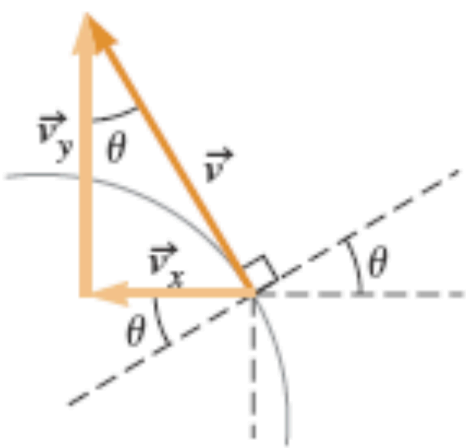


$$|\overrightarrow{r_f}| = |\overrightarrow{r_i}| = r$$

What is the value of the radial acceleration?



Not comfortable with calculus yet? Go back to this when you are and just accept the answers for now



Our object is at position vector $\vec{r}$ at some arbitrary time, with

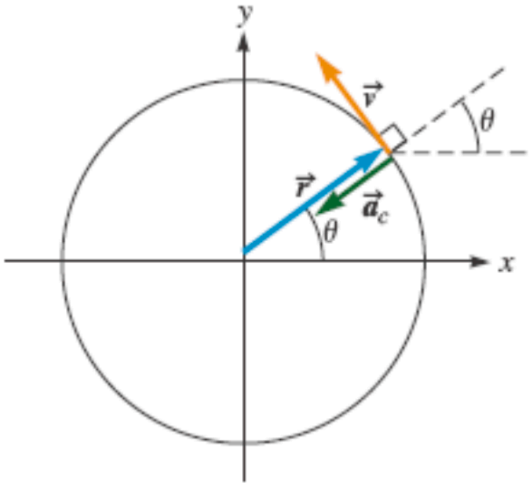
$$x = r\cos\theta, y = r\sin\theta$$

$$\frac{dx}{dt} = v_x = -r\sin\theta\frac{d\theta}{dt} = -r\omega\sin\theta = -v\sin\theta$$

$$\frac{dy}{dt} = v_y = r\cos\theta\frac{d\theta}{dt} = r\omega\cos\theta = v\cos\theta$$

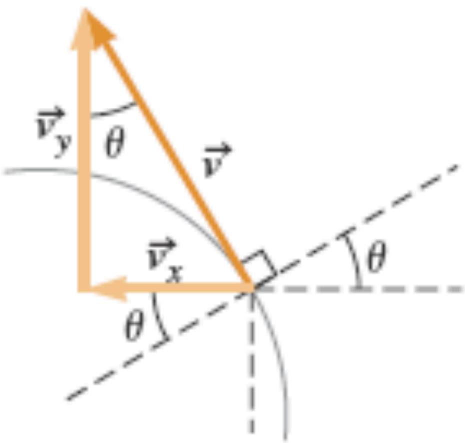
Here only talking about radial acceleration (moving in a circle)

What is the value of the radial acceleration?



$$v_x = -v\sin\theta$$
$$v_y = v\cos\theta$$

$$\vec{v} = (-v\sin\theta)\hat{i} + (v\cos\theta)\hat{j}$$

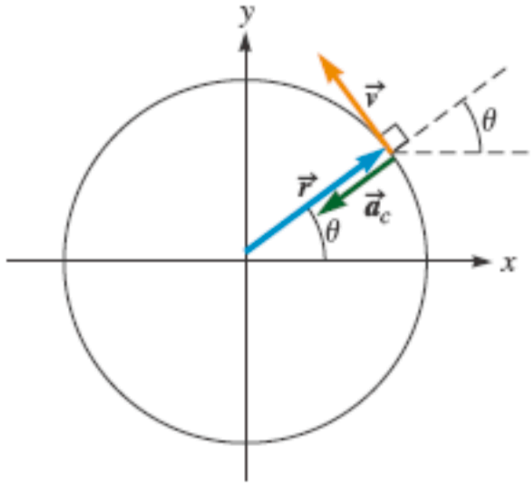


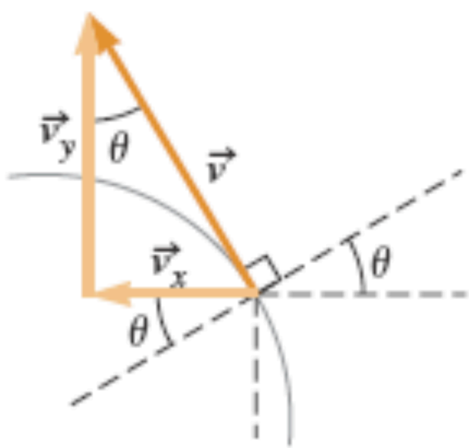
$$\vec{a}_c = \frac{d\vec{v}}{dt} = \frac{d(-v\sin\theta)}{dt}\hat{i} + \frac{d(v\cos\theta)}{dt}\hat{j}$$

$$\vec{a}_c = -v\frac{d(\sin\theta)}{dt}\hat{i} + v\frac{d(\cos\theta)}{dt}\hat{j}$$

Here only talking about radial acceleration (moving in a circle)

What is the value of the radial acceleration?





$$\vec{a}_c = -v\frac{d(\sin\theta)}{dt}\hat{i} + v\frac{d(\cos\theta)}{dt}\hat{j}$$

$$\vec{a}_c = -v\frac{d(\sin\theta)}{d\theta}\frac{d\theta}{dt}\hat{i} + v\frac{d(\cos\theta)}{d\theta}\frac{d\theta}{dt}\hat{j}$$

$$\vec{a}_c = -(v\cos\theta)\left(\frac{d\theta}{dt}\right)\hat{i} - (v\sin\theta)\left(\frac{d\theta}{dt}\right)\hat{j}$$

$$\vec{a}_c = -v\omega\cos\theta\hat{i} - v\omega\sin\theta\hat{j}$$

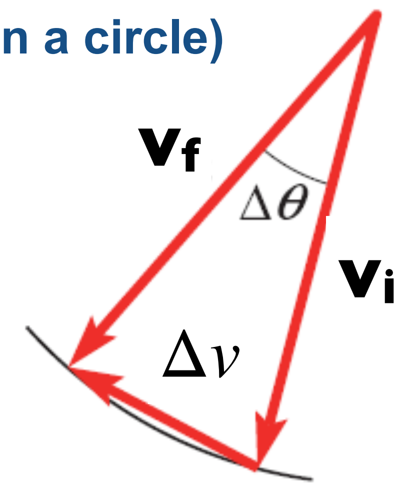
$$\vec{a}_c = -\frac{v^2}{r}\cos\theta\hat{i} - \frac{v^2}{r}\sin\theta\hat{j}$$

Here only talking about radial acceleration (moving in a circle)

SO THE MAGNITUDE OF CENTRIPETAL ACCELERATION IS v^2/r

Connecting circular and linear motion

Here only talking about radial acceleration (moving in a circle)



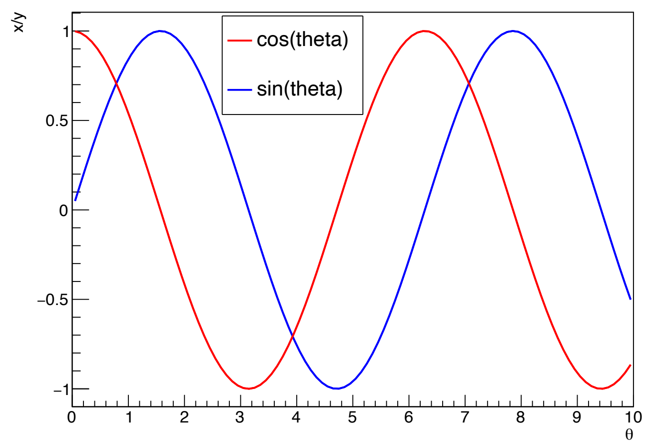
SO THE MAGNITUDE OF CENTRIPETAL ACCELERATION IS v^2/r but …

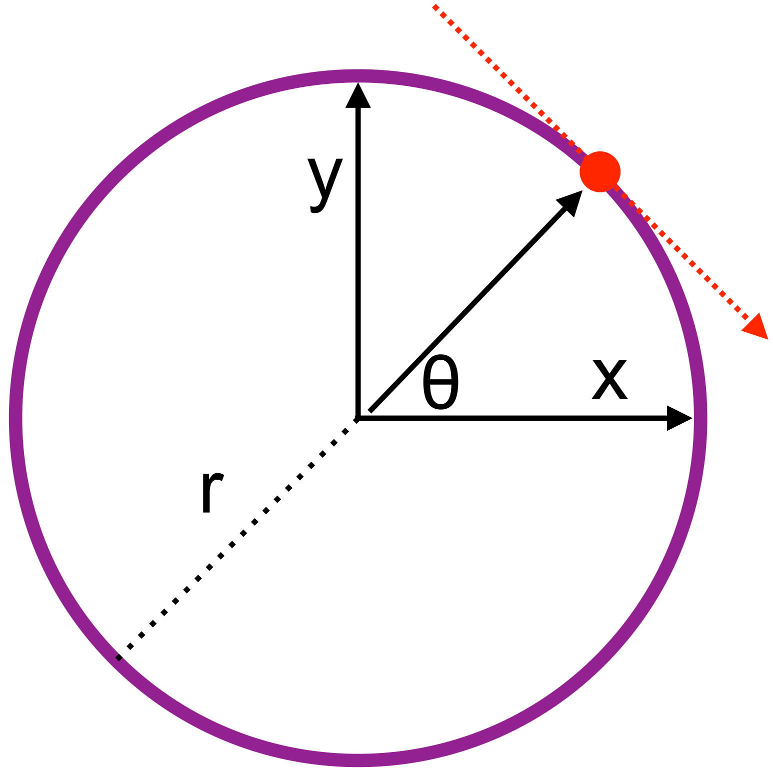
Recall: $v = |\omega| r$

$$a_c = \frac{v^2}{r} = r\omega^2 = v\omega$$

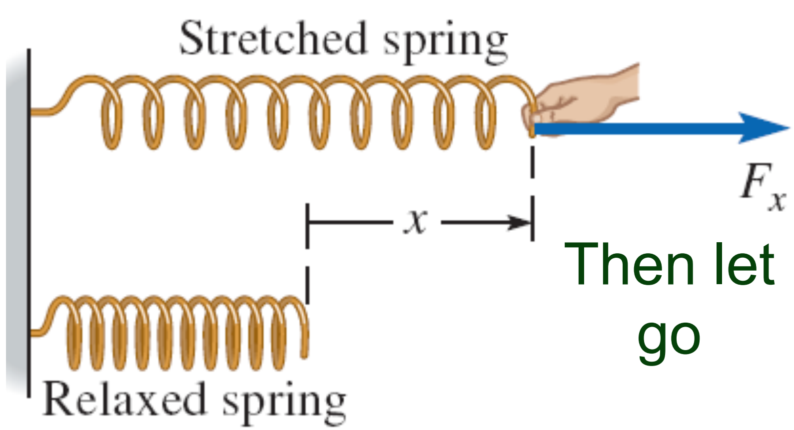
Simple Harmonic Motion

Recall that we have
x = r*cos(θ)
y = r*sin(θ)
and θ=ωt



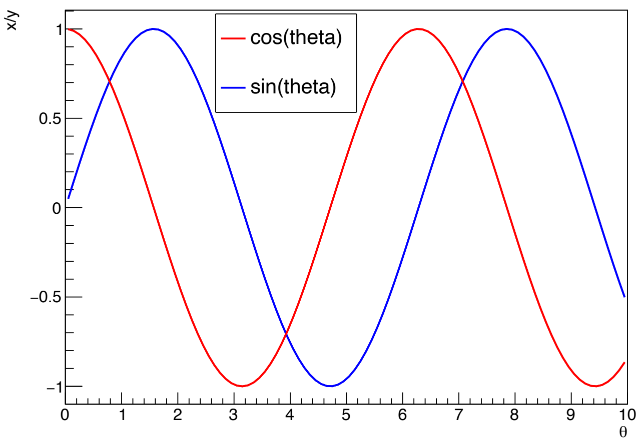


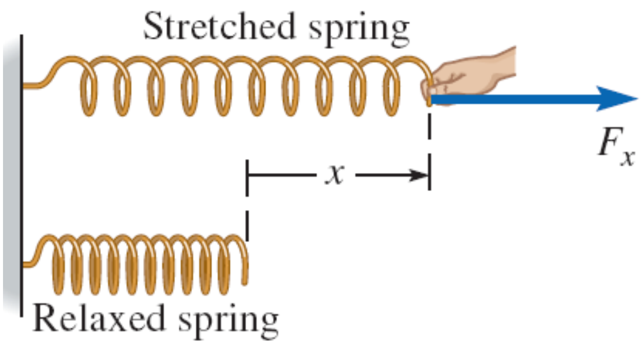
Now let's imagine something different, a spring (we'll talk more about springs in the next chapters). It has some relaxed position, and we pull on it…



Then let go

Simple Harmonic Motion



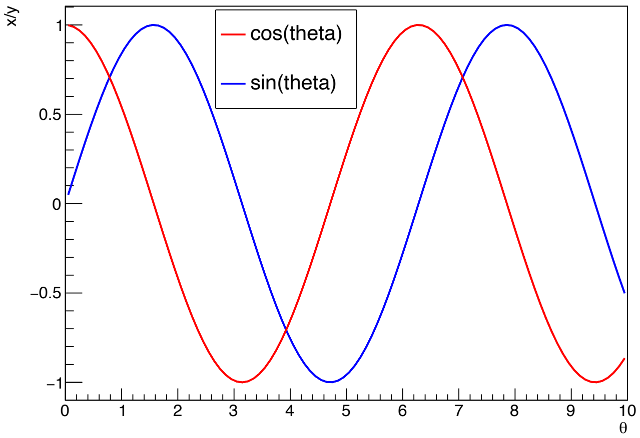


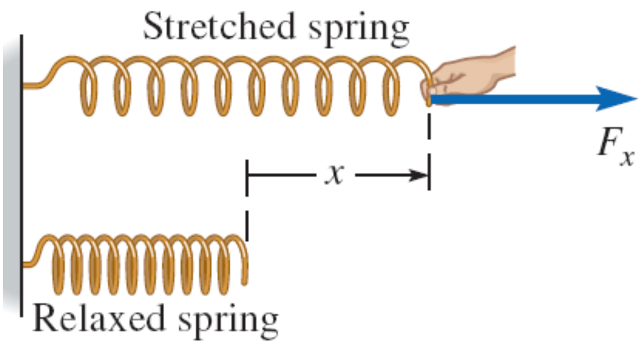
Then let go

What happens to the spring on the left if you pull on it and let go? As we'll see, it tries to return to its natural position. But it will overshoot that position and compress too much. At which point it will want to return back to its natural position again, so this will repeat.

This will continue, and the motion will be periodic, and go as a x=sin(ωt)!

Simple Harmonic Motion



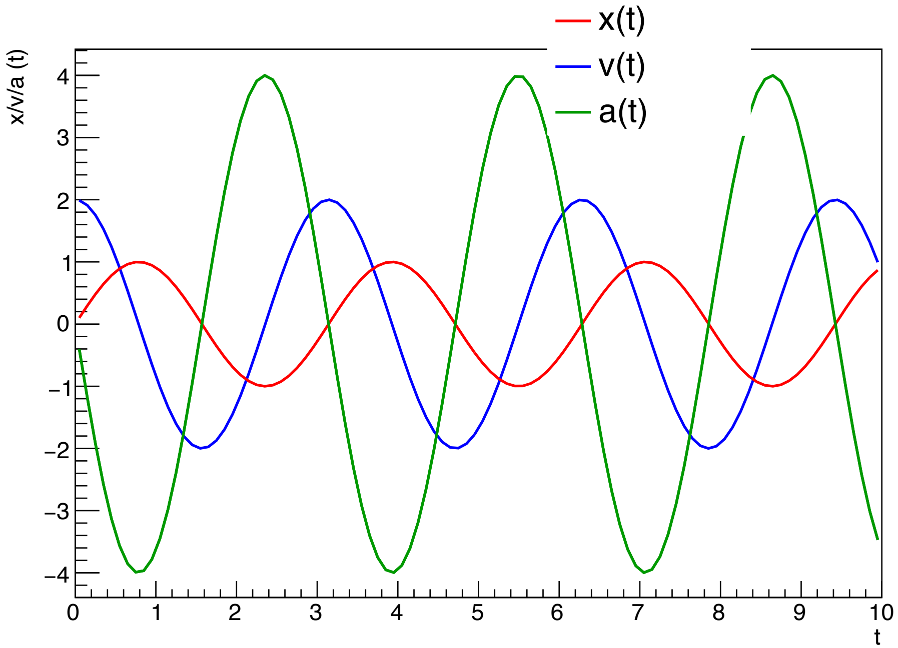


Then let go

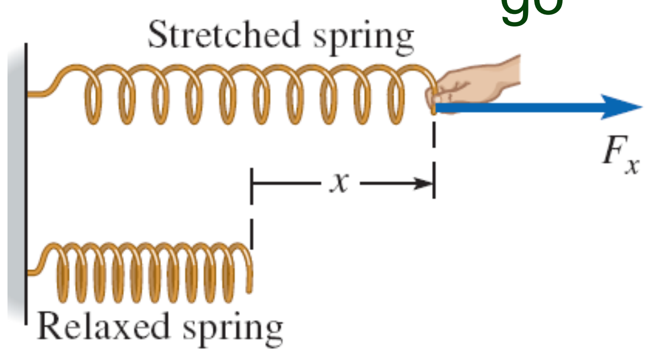
The motion will be periodic, and go as a x=sin(ωt). But we can take the derivative, dx/dt and find that the speed is then v=dx/dt = ωcos(ωt).

Taking the derivative of the speed, we find a=dv/dt = -ω^2sin(ωt)!

Simple Harmonic Motion



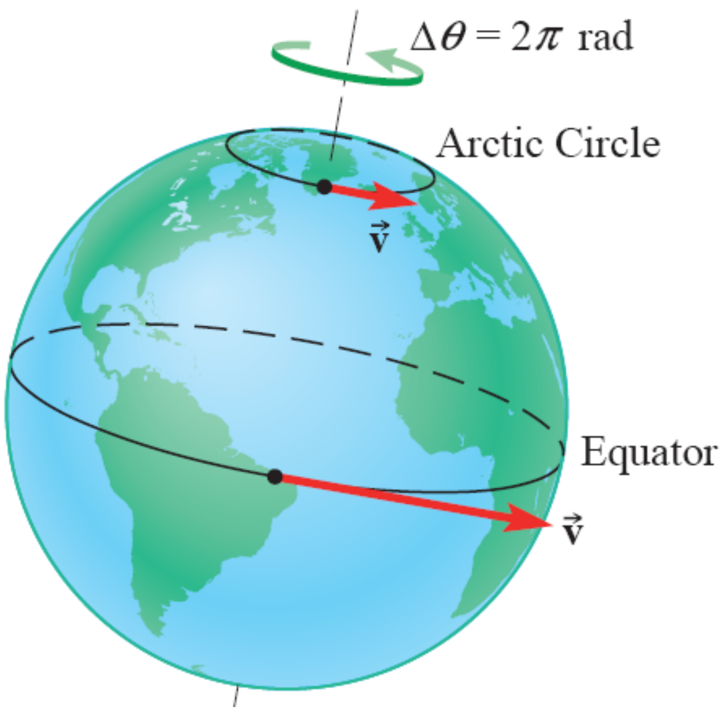
Then let go



$x(t) = \sin(2t)$
$v(t) = 2\cos(2t)$
$a(t) = -4\sin(2t)$

Speed
Acceleration
Position
All have different amplitudes (and sine vs cosine!) but the same frequencies!

Earth is rotating about its axis. What is its angular speed in rad/s? (The question asks for angular *speed*, so we do not have to worry about the direction of rotation.)



What's a convenient time period?

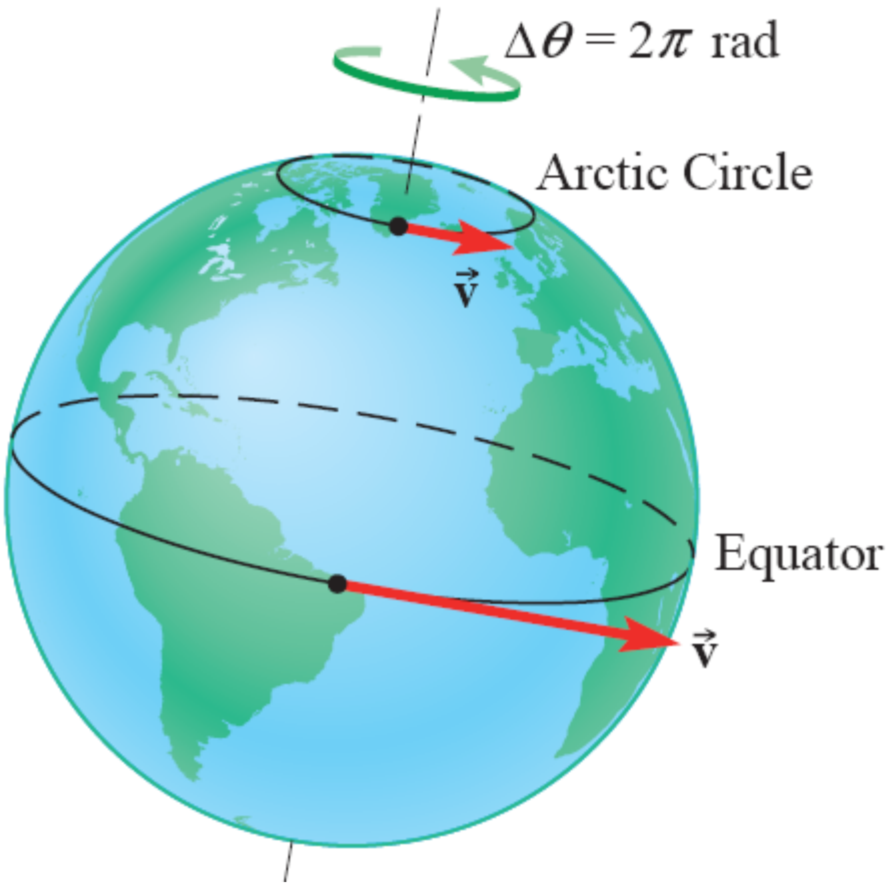
Solution

$$1 \text{ day} = 24 \text{ h} = 24 \text{ h} \times 3600 \text{ s/h} = 86\,400 \text{ s}$$

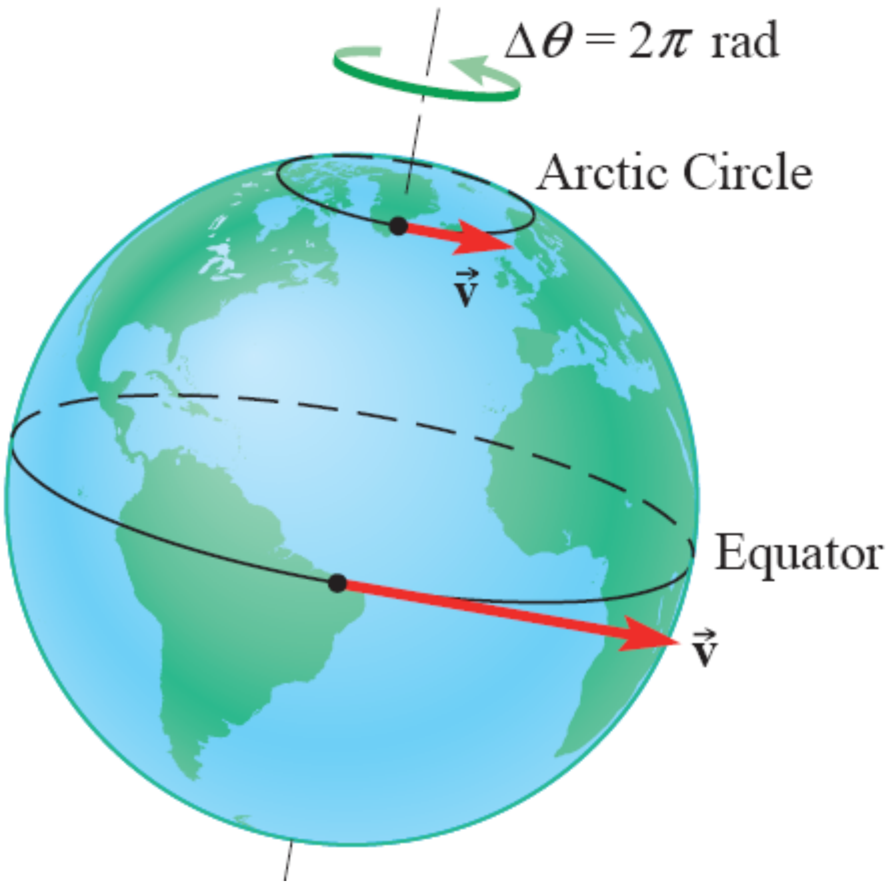
$$|\omega| = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = 7.3 \times 10^{-5} \text{ rad/s}$$

Pretty small angular speed!

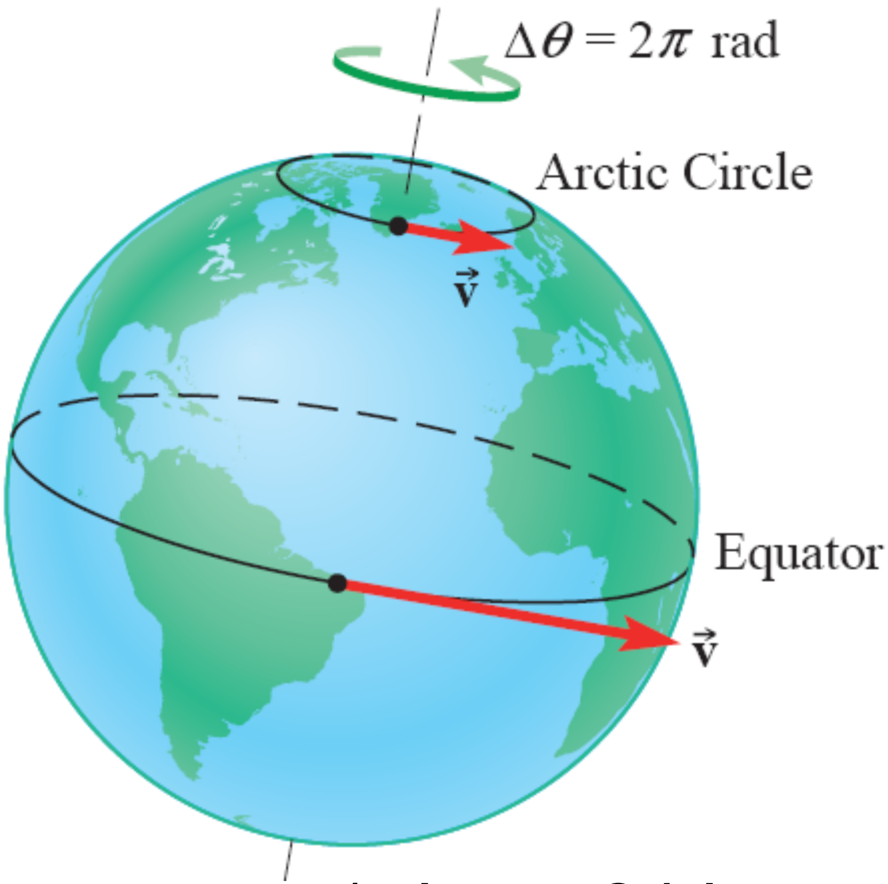
$$v = r|\omega| \quad (\omega \text{ in radians per unit time})$$



A person standing at the equator is moving much faster than another person standing at the Arctic Circle, but their *angular* speeds are the same.

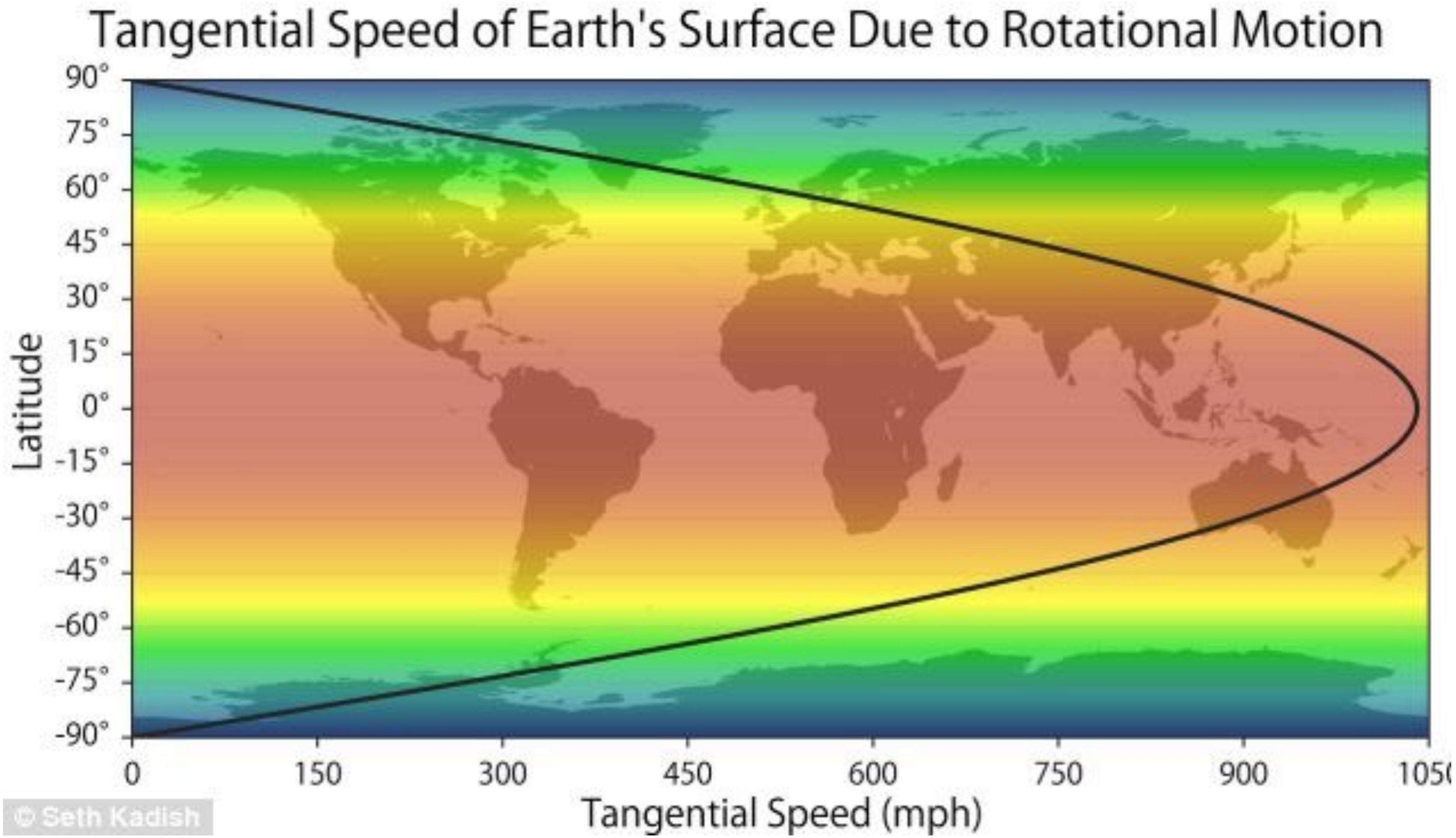


Let's calculate the linear velocity of a person standing on the equator and a person on the Arctic Circle due to the earth spinning. What about someone standing on the South Pole? The radius of the Earth is roughly ~6,370 km



r = 6,370 km = 6.37 x 10^6 m
ω = 1 rot / day
= 2pi rad / day
= 2pi rad * (1 day / 24 hours)
* (1 hour / 60 min) *
(1 min / 60 sec)
ω = 7.3 x 10^{-5} rad/s
v = r ω = 463 m/s!

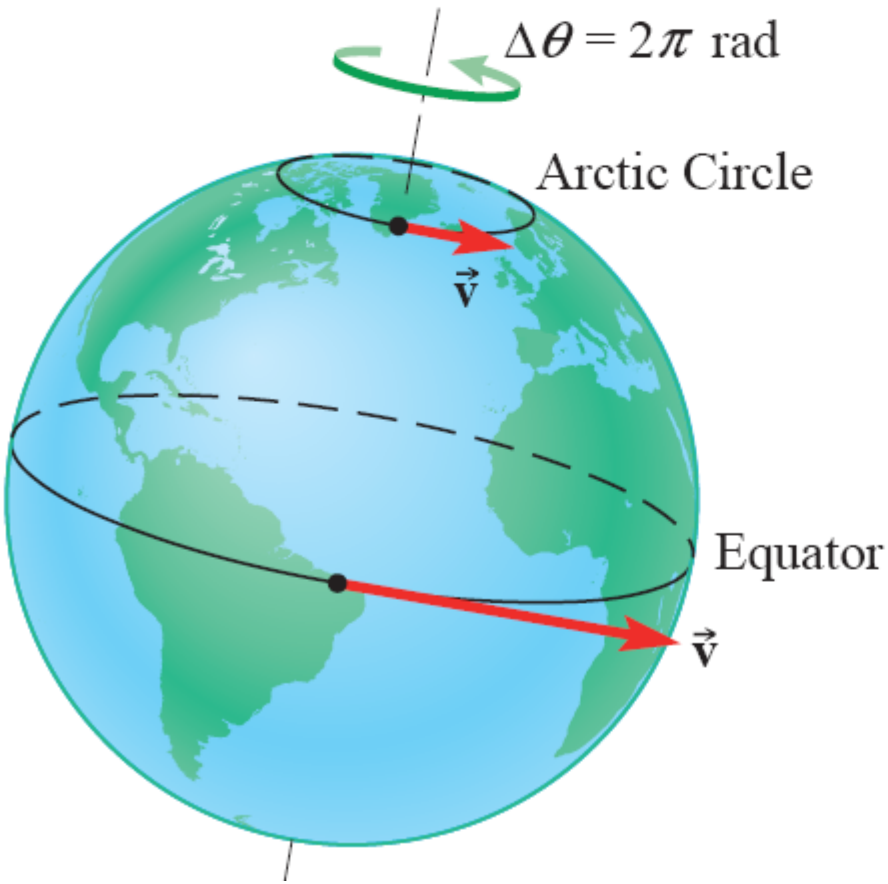
1 day = 24 hours
1 = (1 day / 24 hours) = (24 hours / 1day)



http://www.dailymail.co.uk/sciencetech/article-2546864/How-fast-YOU-spinning-Earths-axis.html

What motion is this ignoring?

What is the centripetal acceleration needed to spin around on the Earth at the equator and North pole?



r(equator) = 6,370 km

= 6.37×10^6 m

r(North pole) = 0!

$\omega = 7.3 \times 10^{-5}$ rad/s

a(equator) = $r\,\omega^2$ = 0.03 m/s^2

a(North pole) = 0

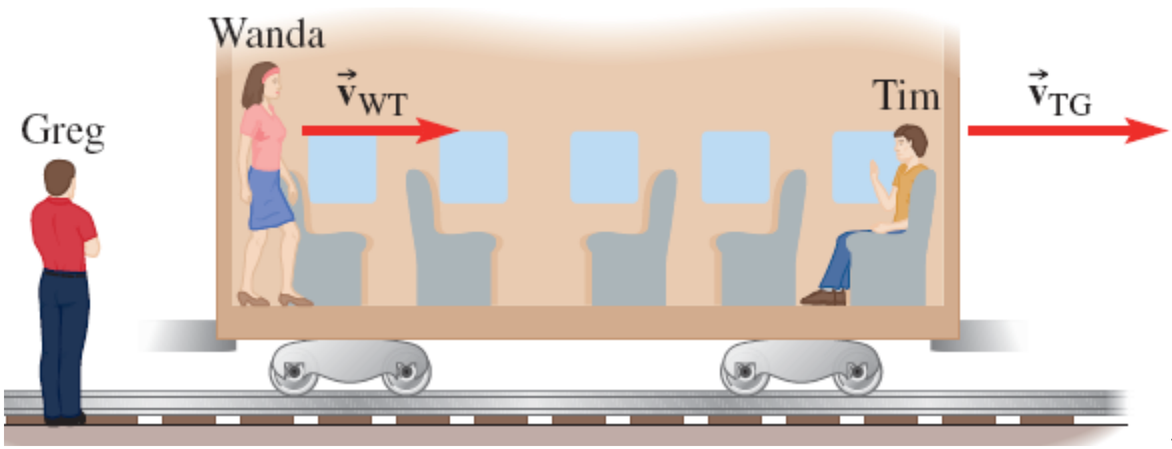
What other acceleration do we know about on Earth and how does this compare?

Relative Velocity

Wanda is walking down the aisle of a train moving along the track at a constant velocity. How fast is Wanda walking? Tim and Greg will give different answers.

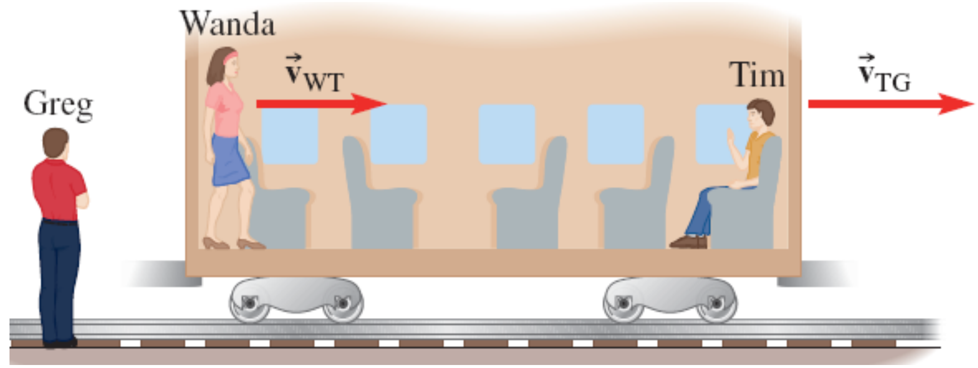
The answer to the question "How fast?" depends on the observer.

WHY? What is special about velocity?



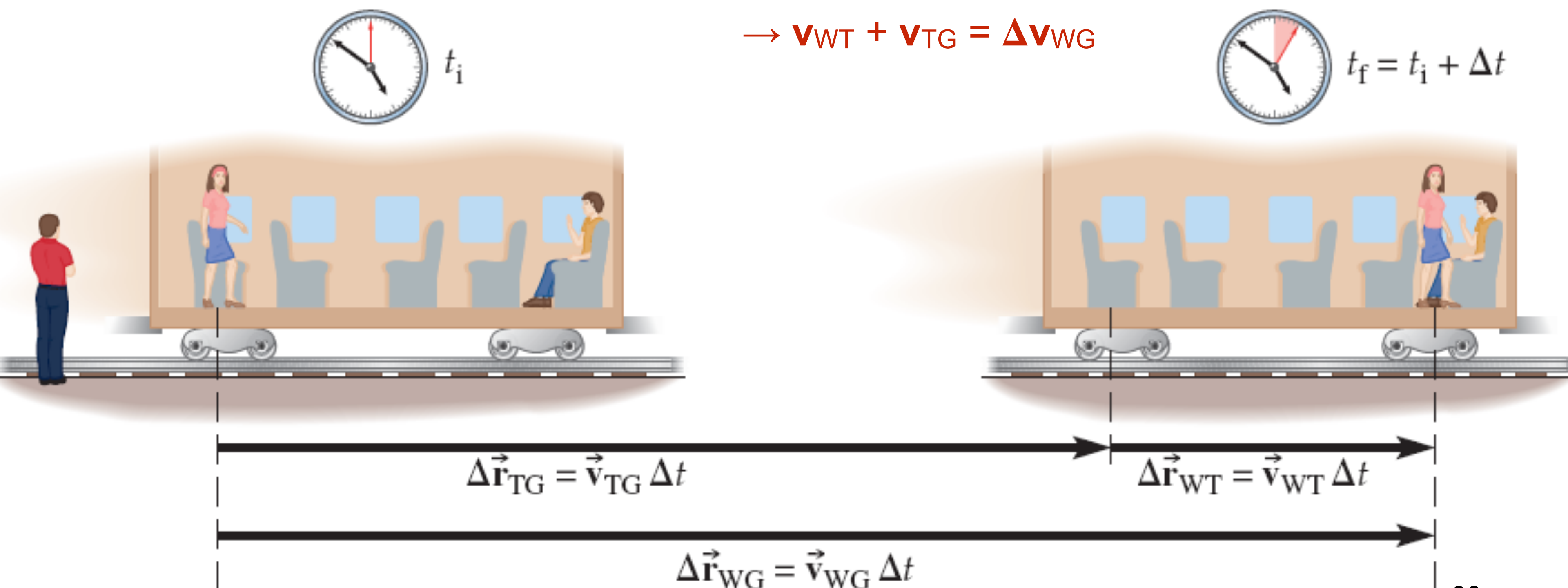
Relative Velocity

Look at this from the lens of vector addition!



$$\Delta\mathbf{r}_{WT} + \Delta\mathbf{r}_{TG} = \Delta\mathbf{r}_{WG} = \mathbf{v}_{WT}\Delta t + \mathbf{v}_{TG}\Delta t = \Delta\mathbf{v}_{WG}\Delta t$$

$$\rightarrow \mathbf{v}_{WT} + \mathbf{v}_{TG} = \Delta\mathbf{v}_{WG}$$



What happens if you have a truck traveling at 60 mph west and it carries a cannon that can fire a ball at 60 mph to the east? (Cool animation/video from Mythbusters)

https://media.wired.com/photos/592744adf3e2356fd800bef6/master/w_582,c_limit/mythbusters-ball-drop-NM.gif

And let's look at what happens when I toss a ball up and down while walking

TIME FOR GROUP PROBLEMS!
https://forms.gle/PiegRLXYJgYe68Hg9

Find the average angular speed of the second hand of an analog clock.

What is its angular displacement during 5.0s?

You throw a 0.15 kg baseball at 35 m/s at an angle of 30.0° above the horizontal. How far does it travel before it hits the ground? What happens if you throw the baseball at 40.0°? 45°? 50°? 60°? Ignore air resistance and the small height above ground at which the ball was released

A glider dives toward the ground at a constant velocity of 4.50 m/s and at an angle of 56 degrees below the horizontal. If the sun is directly overhead, what is the speed of the glider's shadow on the level ground below?

An object moves with an initial velocity = 3.00 jhat m/s and an acceleration = 2.50 ihat m/s^2. Assume the object is initially at the origin.

a) What is the position vector of the object as a function of time?
b) What is the velocity vector of the object as a function of time?
c) What is the position of the object at time t = 3.00 s?
d) What is the speed of the object at time t = 3.00s?

A softball is hit with an initial velocity of 29.0 m/s at an angle of 60 degrees above the horizontal, and impacts the top of the outfield fence 5.00 s later. Assuming the initial height of the softball was 0.500 m above level ground, what are the ball's horizontal and vertical displacements?

A rock is thrown horizontally off a 56.0 m high cliff overlooking the ocean, and the sound of the splash is heard 3.60 s later. If the speed of sound in air at this location is 343 m/s, what was the initial velocity of the rock?

A projectile is launched up and to the right over flat, level ground. If the air resistance is ignored, its maximum range occurs when the angle between its initial velocity and the ground is 45 degrees. Which angles would result in the range being equal to half the maximum?

A child swings a tennis ball attached to a 0.750-m string in a horizontal circle above its head at a rate of 5.00 revolutions/second.

a) What is the centripetal acceleration of the tennis ball?
b) The child now increases the length of the string to 1.00 m, but has to decrease the rate of rotation to 4.00 rev/s. Is the speed of the ball greater now or when the string was shorter?
c) What is the centripetal acceleration of the tennis ball when the string is 1.00 m in length?

Two particles A and B move at a constant speed in circular paths at the same angular speed, ω. Particle A's circle has a radius that is twice the length of particle B's circle.

a) What is the ratio T_A / T_B of their periods?
b) What is the ratio v_A/v_B of their translational speeds?
c) What is the ratio a_A/a_B of the magnitude of their centripetal accelerations?

Pete and Sue, two reckless teenage drivers, are racing eastward along a straight stretch of highway. Pete is traveling at 98.0 km/h, and Sue is chasing him at 125 km/h.

a) What is Pete's velocity with respect to Sue?
b) What is Sue's velocity with respect to Pete?
c) If Sue is initially 325 m behind Pete, how long will it take her to catch up to him?

Suppose at one point along the Nile River, a ferryboat must travel straight across a 10.3 mile stretch from west to east. At this location, the river flows from south to north with a speed of 2.41 m/s. The ferryboat has a motor that can move the boat forward at a constant speed of 20.0 mph in the water.

a) In what direction should the ferry captain direct the boat so as to travel directly across the river?
b) If the captain instead points the boat directly at the target location on the east bank, how far downstream will she be from the target when she hits land?

Truck A is traveling northward along a straight stretch of highway at 110 km/hr. Truck B is traveling southward in the adjacent lane at 90 km/hr. If the trucks are initially 1190m apart, how many seconds will it take for them to be next to one another?

Centrifuges are used to establish the maximum acceleration a pilot can withstand without “blacking out”.

If the pilot undergoes a radial acceleration of 4.00 *g* (as measured at her head) and the radial distance from her head to the axis of rotation is 12.5 m, what is the period of rotation of the centrifuge?

An airplane flies from Denver to Chicago (1770 km) in 4.4 h when no wind blows.

On a day with a tailwind, the plane makes the trip in 4.0 h.

(a) What is the wind speed?
(b) If a headwind blows with the same speed, how long does the trip take?

Ever notice that trips east on an airplane typically take less time than the same trips, but in reverse, to the west?

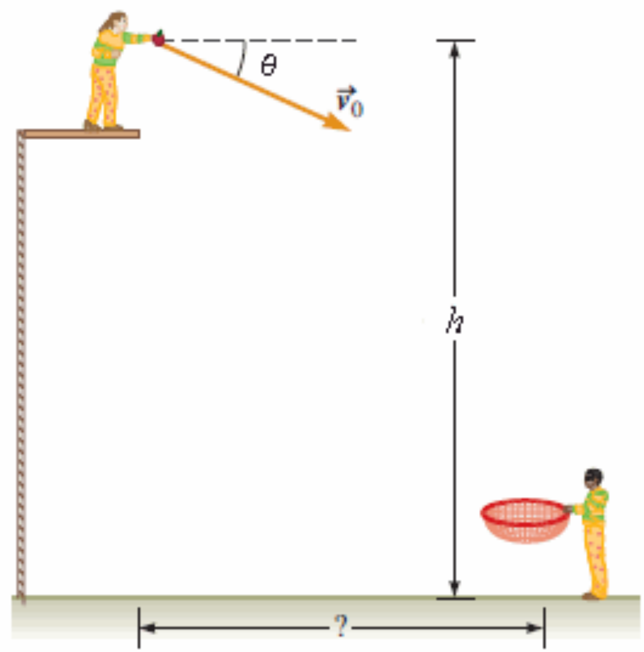
Jack wants to row directly across a river from the east shore to a point on the west shore.

The width of the river is 250 m and the current flows from north to south at 0.61 m/s. The trip takes Jack 4.2 min.

In what direction did he head his rowboat to follow a course due west across the river?

At what speed with respect to still water is Jack able to row?

A circus performer stands on a platform and throws an apple from a height h=45 m above the ground with an initial velocity **v_0** as shown in the figure. A second, blindfolded performer must catch the apple. If v_0 = 26 m/s and θ=30 degrees, how far from the end of the platform should the second performer stand?



A circus performer is shot out of a cannon and flies over a net that is placed horizontally 6.0 m from the cannon. When the cannon is aimed at an angle of 40 degrees above the horizontal, the performer is moving in the horizontal direction and just barely clears the net as she passes over it.

a) What is the muzzle speed of the cannon?
b) How high is the net?